## Problem 1.2

The Rankine temperature scale (abbreviated ${ }^{\circ} \mathrm{R}$ ) uses the same size degrees as Fahrenheit, but measured up from absolute zero like kelvin (so Rankine is to Fahrenheit as kelvin is to Celsius). Find the conversion formula between Rankine and Fahrenheit, and also between Rankine and kelvin. What is room temperature on the Rankine scale?

## Solution

Formulas converting between temperature scales are linear functions, so the conversion formula from Fahrenheit to Rankine has the form,

$$
\left({ }^{\circ} \mathrm{R}\right)=m\left({ }^{\circ} \mathrm{F}\right)+b,
$$

where $m$ and $b$ are constants to be determined. The Rankine temperature scale uses the same size degrees as Fahrenheit, which means $m=1$.

$$
\left({ }^{\circ} \mathrm{R}\right)=\left({ }^{\circ} \mathrm{F}\right)+b
$$

In order to find $b$, use the fact that at absolute zero the Rankine and Fahrenheit temperatures are $0{ }^{\circ} \mathrm{R}$ and $-459.67^{\circ} \mathrm{F}$, respectively.

$$
0=-459.67+b \quad \rightarrow \quad b=459.67
$$

Therefore, the conversion formula between Rankine and Fahrenheit is

$$
\left({ }^{\circ} \mathrm{R}\right)=\left({ }^{\circ} \mathrm{F}\right)+459.67 .
$$

Similarly, the conversion formula from kelvin to Rankine has the form,

$$
\left({ }^{\circ} \mathrm{R}\right)=M(\mathrm{~K})+B,
$$

where $M$ and $B$ are constants to be determined. Both the Rankine and kelvin temperatures are zero at absolute zero.

$$
0=M(0)+B \quad \rightarrow \quad B=0
$$

As a result,

$$
\left({ }^{\circ} \mathrm{R}\right)=M(\mathrm{~K}) .
$$

Use the fact that water freezes at $0{ }^{\circ} \mathrm{C}$ (that is, $0+273.15 \mathrm{~K}$ ) and $32{ }^{\circ} \mathrm{F}$ (that is, $32+459.67^{\circ} \mathrm{R}$ ) to determine $M$.

$$
491.67=M(273.15) \quad \rightarrow \quad M=\frac{491.67}{273.15}=\frac{9}{5}
$$

Therefore, the conversion formula between Rankine and kelvin is

$$
\left({ }^{\circ} \mathrm{R}\right)=\frac{9}{5}(\mathrm{~K}) .
$$

Room temperature is $25{ }^{\circ} \mathrm{C}$, which is $25+273.15 \mathrm{~K}$. Therefore, room temperature on the Rankine scale is

$$
\left({ }^{\circ} \mathrm{R}\right)=\frac{9}{5}(25+273.15)=536.67^{\circ} \mathrm{R} .
$$

